

Series: Mathematics - from zero to engineer (10).

Number systems - the decimal number system

	2	7	6	5	1	3	2	3
place values	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	
	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	

- the place values are powers of 10, hence the decimal number system.

The binary number system (base 2)

	1	0	1	1	1	0	1
place values	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

- the place values are powers of 2.

Converting a binary number to a decimal number.

a) $1011,101_2 = 11,625_{10}$

$$1 \cdot 8 + 0 \cdot 4 + 1 \cdot 2 + 1 \cdot 1 + 1 \cdot \frac{1}{2} + 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{8} = 11 + \frac{1}{2} + \frac{1}{8} = 11\frac{5}{8} = 11,625$$

b) $1101,011_2 = 13,375_{10}$

$$1 \cdot 8 + 1 \cdot 4 + 0 \cdot 2 + 1 \cdot 1 + 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{4} + 1 \cdot \frac{1}{8} = 13 + \frac{1}{4} + \frac{1}{8} = 13\frac{3}{8} = 13,375$$

- the small subscripts 2 and 10 indicate the bases of the two number systems.

The octal number system (base 8).

- the octal number system uses the following symbols: 0, 1, 2, 3, 4, 5, 6, 7.

	3	5	7	1	3	2	1
place values	8^2	8^1	8^0	8^{-1}	8^{-2}	8^{-3}	
	64	8	1	$\frac{1}{8}$	$\frac{1}{64}$	$\frac{1}{512}$	

- the place values are powers of 8.

Converting an octal number to a decimal number.

a) $357,321_8 = 239,408_{10}$

$$3 \cdot 64 + 5 \cdot 8 + 7 \cdot 1 + 3 \cdot \frac{1}{8} + 2 \cdot \frac{1}{64} + 1 \cdot \frac{1}{512} = 192 + 40 + 7 + \frac{3}{8} + \frac{1}{32} + \frac{1}{512} =$$

$$= 239 \frac{209}{512} = 239,408 \text{ (correct to three decimal places).}$$

b) $263,452_8 = 179,582_{10}$

$$2 \cdot 64 + 6 \cdot 8 + 3 \cdot 1 + 4 \cdot \frac{1}{8} + 5 \cdot \frac{1}{64} + 2 \cdot \frac{1}{512} = 128 + 48 + 3 + \frac{1}{2} + \frac{5}{64} + \frac{1}{256} =$$

$$= 179 \frac{149}{256} = 179,582 \text{ (correct to three decimal places).}$$

The duodecimal number system (base 12)

- the duodecimal number system uses the following symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, X, A.

	2	X	5	,	1	3	6
place values	12^2	12^1	12^0		12^{-1}	12^{-2}	12^{-3}
	144	12	1		$\frac{1}{12}$	$\frac{1}{144}$	$\frac{1}{1728}$

- the place values are powers of 12.

Converting a duodecimal number to a decimal number.

a) $2X5,136_{12} = 413,108_{10}$

$$2 \cdot 144 + 10 \cdot 12 + 5 \cdot 1 + 1 \cdot \frac{1}{12} + 3 \cdot \frac{1}{144} + 6 \cdot \frac{1}{1728} = 288 + 120 + 5 + \frac{1}{12} + \frac{3}{144} + \frac{1}{288} =$$

$$413 \frac{31}{288} = 413,108 \text{ (correct to three decimal places).}$$

b) $3A4,265_{12} = 568,211_{10}$

$$3 \cdot 144 + 11 \cdot 12 + 4 \cdot 1 + 2 \cdot \frac{1}{12} + 6 \cdot \frac{1}{144} + 5 \cdot \frac{1}{1728} = 432 + 132 + 4 + \frac{1}{6} + \frac{1}{24} + \frac{5}{1728} =$$

$$568 \frac{365}{1728} = 568,211 \text{ (correct to three decimal places).}$$

The hexadecimal number system.

- the hexadecimal number system uses the following symbols:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

	2	A	7	,	3	E	2
place values	16^2	16^1	16^0		16^{-1}	16^{-2}	16^{-3}
	256	16	1		$\frac{1}{16}$	$\frac{1}{256}$	$\frac{1}{4096}$

- the place values are powers of 16.

Converting a hexadecimal number to a decimal number.

$$a) 2A7,3E2_{16} = 679,243_{10}$$

$$2 \cdot 256 + 10 \cdot 16 + 7 \cdot 1 + 3 \cdot \frac{1}{16} + 14 \cdot \frac{1}{256} + 2 \cdot \frac{1}{4096} = 512 + 160 + 7 + \frac{3}{16} + \frac{7}{128} + \frac{1}{2048} = 679 \frac{497}{2048} = 679,243 \text{ (correct to three decimal places).}$$

$$b) 3C4,21F_{16} = 964,133_{10}$$

$$3 \cdot 256 + 12 \cdot 16 + 4 \cdot 1 + 2 \cdot \frac{1}{16} + 1 \cdot \frac{1}{256} + 15 \cdot \frac{1}{4096} = 768 + 192 + 4 + \frac{1}{8} + \frac{1}{256} + \frac{15}{4096} = 964 \frac{543}{4096} = 964,133 \text{ (correct to three decimal places).}$$