

Series: Mathematics - from zero to engineer (19).

Algebra: Review exercises.

Simplify each of the following expressions by combining like terms.

$$a) 4xy + 3xz - 6zy - 5zx + yx = 5xy - 2zx - 6zy$$

$$b) -2a + 4ab + a - 4ba = -a$$

$$c) 3rst - 10str + 8ts - 5rt + 2st = -7rst + 10st - 5rt$$

$$d) 2pq - 4pr + qr - 2rq + 3qp = 5pq - 4pr - qr$$

$$e) 5lmn - 6ml + 7lm + 8mnl - 4ln = 13lmn + lm - 4ln$$

Simplify each of the following expressions by combining like terms and factoring out the common factor.

$$a) 4xy + 3xz - 6zy - 5zx + yx = 5xy - 2zx - 6zy = x(5y - 2z) - 6zy = \\ = 5xy - z(2x + 6y) = y(5x - 6z) - 2zx$$

$$b) 3rst - 10str + 8ts - 5rt + 2st = -7rst + 10st - 5rt = t(-7rs + 10s - 5r) = \\ = st(-7r + 10) - 5rt = rt(-7s - 5) + 10st$$

$$c) 2pq - 4pr + qr - 2rq + 3qp = 5pq - 4pr - qr = p(5q - 4r) - qr = \\ = 5pq - r(4p + q) = q(5p - r) - 4pr$$

$$d) 5lmn - 6ml + 7lm + 8mnl - 4ln = 13lmn + lm - 4ln = l(13mn + m - 4n) = \\ = lm(13n + 1) - 4ln = ln(13m - 4) + lm.$$

Expand the following expressions, then factorise them where possible.

$$a) 8x(y - z) + 2y(7x + z) = 8xy - 8xz + 14xy + 2yz = 22xy - 8xz + 2yz = \\ = 2[x(11y - 4z) + yz]$$

$$b) (3a - b)(b - 3a) + b^2 = 3ab - 9a^2 - \cancel{b^2} + 3ab + \cancel{b^2} = 6ab - 9a^2 = \\ = 3a(2b - 3a).$$

$$c) -3\{w - 7[x - 8(3 - z)]\} = -3\{w - 7[x - 24 + 8z]\} = -3\{w - 7x + 168 - 56z\} = \\ = -3w + 21x - 504 + 168z$$

$$d) \frac{2a - 3}{4b} + \frac{3a + 2}{6b} = \frac{6a - 9 + 6a + 4}{12b} = \frac{12a - 5}{12b} = \frac{a}{b} - \frac{5}{12b} = \frac{1}{12b}(12a - 5)$$

Powers.

coefficient $\leftarrow 5a^3 \rightarrow$ exponent
base

The laws of exponents.

$$1) a^m \cdot a^n = a^{m+n}$$

$$2) a^m : a^n = a^{m-n}$$

$$3) (a^m)^n = a^{m \cdot n}$$

$$4) a^0 = 1$$

$$5) a^{-m} = \frac{1}{a^m}$$

$$6) a^{\frac{1}{m}} = \sqrt[m]{a}$$

$$7) a^{\frac{n}{m}} = (\sqrt[m]{a})^n = \sqrt[m]{a^n}$$

$$8) (ab)^n = a^n \cdot b^n$$

$$9) \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\frac{6x^{-4} \cdot 2x^3}{8x^{-3}} = \frac{6 \cdot 2x^{-4+3}}{8x^{-3}} = \frac{12x^{-1}}{8x^{-3}} = \frac{12}{8} x^{-1-(-3)} = \frac{3}{2} x^{-1+3} = \frac{3}{2} x^2$$

$$E = (5x^2y^{-\frac{3}{2}}2^{\frac{1}{4}})^2 \cdot (4x^4y^2z)^{-\frac{1}{2}} = 25x^{2 \cdot 2}y^{-\frac{3}{2} \cdot 2}2^{\frac{1}{4} \cdot 2} \cdot 4^{-\frac{1}{2}}x^{4 \cdot (-\frac{1}{2})}y^{2 \cdot (-\frac{1}{2})}z^{-\frac{1}{2}} =$$

$$= 25x^4y^{-3}2^{\frac{1}{2}} \cdot \frac{1}{2}x^{-2}y^{-1}z^{-\frac{1}{2}} = \frac{25}{2}x^{4+(-2)}y^{-3+(-1)}z^{\frac{1}{2}+(-\frac{1}{2})} = \frac{25}{2}x^2y^{-4}z^0 = \frac{25}{2}x^2y^{-4} \cdot 1$$

$$E = \frac{25}{2}x^2y^{-4} = \frac{25x^2}{2y^4}$$

$$F = \sqrt[3]{a^6b^3} : \sqrt{\frac{1}{9}a^4b^6} \cdot (4\sqrt{a^6b^2})^{-\frac{1}{2}} = (a^6b^3)^{\frac{1}{3}} : (\frac{1}{9}a^4b^6)^{\frac{1}{2}} \cdot [4(a^6b^2)^{\frac{1}{2}}]^{-\frac{1}{2}} =$$

$$= a^{6 \cdot \frac{1}{3}}b^{3 \cdot \frac{1}{3}} : \frac{1}{3}a^{4 \cdot \frac{1}{2}}b^{6 \cdot \frac{1}{2}} \cdot (4a^{6 \cdot \frac{1}{2}}b^{2 \cdot \frac{1}{2}})^{-\frac{1}{2}} = a^2b : \frac{1}{3}a^2b^3 \cdot (4a^3b)^{-\frac{1}{2}} =$$

$$= 3b^{-2} \cdot \frac{1}{2}a^{3 \cdot (-\frac{1}{2})}b^{-\frac{1}{2}} = \frac{3}{2}a^{-\frac{3}{2}}b^{-\frac{5}{2}} = \frac{3}{2\sqrt{a^3b^5}} = \frac{3}{2\sqrt{a^2ab^4b}} = \frac{3}{2ab^2\sqrt{ab}}$$