Series: Mathematics - from zero to engineer (20).

Logarithms.

$$b = a^c, \quad a > 0, a \neq 1, b > 0$$

$c = \log_a b \rightarrow$ c is the logarithm of b to the base a.

a - base

b - argument (the number whose logarithm is being taken)

c - result (exponent)

a logarithm is the exponent to which a given base must be raised to obtain a given number.

a) $x = \log_2 16 \quad 2^x = 16 \quad x = 4$

b) $4 = \log_x 81 \quad x^4 = 81 \quad x = 3$

c) $2 = \log_7 x \quad 7^2 = x \quad x = 49$

The laws of logarithms.

1) $\log_a(xy) = \log_a x + \log_a y$ the logarithm of a product is equal to the sum of the logarithms.

2) $\log_a(x:y) = \log_a x - \log_a y$ the logarithm of a quotient is equal to the difference of the logarithms.

3) $\log_a x^n = n \cdot \log_a x$ the logarithm of a number raised to a power is equal to the exponent multiplied by the logarithm of the number.

4) $\log_a 1 = 0$

5) $\log_a a = 1$

6) $\log_a a^x = x$

7) $a^{\log_a x} = x$

8) $\frac{1}{\log_b a} = \log_a b$

a) $\log_a(6{,}788 \cdot 1{,}043) = \log_a 6{,}788 + \log_a 1{,}043$

b) $\log_a(19{,}112 : 0{,}054) = \log_a 19{,}112 - \log_a 0{,}054$

c) $\log_a 5{,}889^{1{,}2} = 1{,}2 \cdot \log_a 5{,}889$

d) $\log_8 1 = 0$

e) $\log_7 7 = 1$

f) $\log_3 27 = 3$

g) $12^{\log_{12} 4} = 4$

h) $\frac{1}{\log_3 4} = \log_4 3$

Common logarithm ($\log_{10} a$ / $\log a$) and natural logarithm ($\log_e a$ / $\ln a$)

- the number e (Euler's number, Napier's constant, the base of the natural logarithm) → $e \approx 2,71828...$

Calculate the value of each of the following logarithms, correct to three decimal places (use a calculator).

a) $\log 5,321 = 0,726$

b) $\log 0,278 = -0,556$

c) $\log 1 = 0$

d) $\log(-1,005)$ → it cannot be evaluated in the set of real numbers.

e) $\ln 13,45 = 2,599$

f) $\ln 0,278 = -1,280$

g) $\ln 0,00001 = -11,513$

h) $\ln(-0,001)$ → it cannot be evaluated in the set of real numbers.

Change of base of a logarithm.

$\log_a x = \log_a 10 \cdot \log_{10} x$

a) $\log_2 3,66 = 1,872$

$\log_2 10 \cdot \log_{10} 3,66 = 3,322 \cdot 0,563 = 1,872$

b) $\log_{3,4} 0,293 = -1,003$

$\log_{3,4} 10 \cdot \log_{10} 0,293 = 1,882 \cdot (-0,533) = -1,003$

c) $\log_{9,9} 6,35 = 0,806$

$\log_{9,9} 10 \cdot \log_{10} 6,35 = 1,004 \cdot 0,803 = 0,806$

d) $\log_{7,34} 7,34 = 1.$