Series: Mathematics – from zero to engineer (21).

Logarithmic equations.

Simplify.

$$2\log_a x - 3\log_a(2x) + \log_a x^2 = \log_a x^2 - \log_a(2x)^3 + \log_a x^2 =$$

$$= \log_a x^2 - \log_a 8x^3 + \log_a x^2 = \log_a \frac{x^2}{8x^3}\cdot x^2 = \log_a \frac{x^4}{8x^3} = \log_a \frac{x}{8} = \log_a \frac{1}{8}x$$

Solve the following equation for x.

$$4\log_a \sqrt{x} - \log_a(3x) = \log_a x^{-2}$$

$$\log_a (x^{\frac{1}{2}})^4 - \log_a(3x) = \log_a x^{-2}$$

$$\log_a x^2 - \log_a(3x) = \log_a x^{-2}$$

$$\frac{x^2}{3x} = \frac{1}{x^2} \qquad \frac{x}{3} = \frac{1}{x^2} \qquad x^3 = 3 \qquad x = \sqrt[3]{3}$$

Find y in terms of x, given that:

$$2\log_a y - 3\log_a(x^2) = \log_a \sqrt{y} + \log_a x$$

$$\log_a y^2 - \log_a (x^2)^3 = \log_a \sqrt{y} + \log_a x$$

$$\log_a y^2 - \log_a \sqrt{y} = \log_a x + \log_a x^6$$

$$\frac{y^2}{\sqrt{y}} = x \cdot x^6 \qquad \frac{y^2}{y^{\frac{1}{2}}} = x^7 \qquad y^{\frac{3}{2}} = x^7$$

$$y = (x^7)^{\frac{2}{3}}$$

$$y = x^{\frac{14}{3}}$$

$$y = \sqrt[3]{x^{14}}$$

Review exercises

Simplify the following expressions:

a) $a^6 \cdot a^5 = a^{11}$

b) $x^7 : x^3 = x^4$

c) $(w^2)^m : (w^m)^3 = w^{2m} : w^{3m} = w^{-m}$

d) $s^3 : t^{-4} \cdot (s^{-3}t^{-2})^3 = s^3 : t^{-4} \cdot s^{-9} \cdot t^{-6} = s^3 \cdot t^4 \cdot s^{-9} \cdot t^{-6} = s^{-6}t^{-2}$

e) $\frac{8x^{-3} \cdot 3x^2}{6x^{-4}} = \frac{24x^{-1}}{6x^{-4}} = 4x^3$

f) $(4a^3b^{-1}c)^2 \cdot (a^{-2}b^4c^{-2})^{\frac{1}{2}} : [64(a^6b^4c^2)^{-\frac{1}{2}}] =$

$= 16a^6b^{-2}c^2 \cdot a^{-1}b^2c^{-1} : (64a^{-3}b^{-2}c^{-1}) =$

$= \frac{16a^5b^0c}{64a^{-3}b^{-2}c^{-1}} = \frac{1}{4}a^8b^2c^2$

g) $\sqrt[3]{8a^3b^6} : \sqrt{\frac{1}{25}a^4b^7} \cdot (16\sqrt{a^4b^6})^{-\frac{1}{2}} =$

$= (8a^3b^6)^{\frac{1}{3}} : \left(\frac{1}{25}a^4b^7\right)^{\frac{1}{2}} \cdot (16a^2b^3)^{-\frac{1}{2}} =$

$= 2ab^2 : \left(\frac{1}{5}a^2b^{\frac{7}{2}}\right) \cdot \frac{1}{4}a^{-1}b^{-\frac{3}{2}} =$

$= 2ab^2 \cdot 5a^{-2}b^{-\frac{7}{2}} \cdot \frac{1}{4}a^{-1}b^{-\frac{3}{2}} =$

$= \frac{5}{2}a^{-2}b^{-3}$

Rewrite the following expressions without using logarithms.

a) $\log K = \log P - \log T + 1{,}3 \log V$

$\log K = \log \frac{P}{T} + \log V^{1,3}$

$\log K = \log \frac{PV^{1,3}}{T} \longrightarrow K = \frac{PV^{1,3}}{T}$

b) $\ln A = \ln P + m$

$m = \ln A - \ln P$

$m = \ln \frac{A}{P} \longrightarrow \frac{A}{P} = e^m \,/\cdot P \quad A = Pe^m$

Write $R = r\sqrt{\frac{f+P}{f-P}}$ in logarithmic form.

$R = r\frac{\sqrt{f+P}}{\sqrt{f-P}} \longrightarrow \log R = \log\left(r\frac{\sqrt{f+P}}{\sqrt{f-P}}\right)$

$\log R = \log r + \log\sqrt{f+P} - \log\sqrt{f-P}$

Evaluate using a calculator or, where necessary, by applying the change-of-base formula (correct to three decimal places).

a) $\log 5,324 = 0,726$

b) $\ln 0,0023 = -6,075$

c) $\log_4 1,2 = 0,132$

$$\log_4 10 \cdot \log_{10} 1,2 = \frac{\log_{10} 1,2}{\log_{10} 4} = \frac{0,079}{0,602} = 0,132$$