

Series: Mathematics - from zero to engineer (24)

Factoring algebraic expressions.

Common factors.

$$a) 8x^4y^3 + 6x^3y^2 = 2x^3y^2(4xy + 3)$$

$$b) 15a^3b - 9a^2b^2 = 3a^2b(5a - 3b)$$

Common factors obtained by grouping (factoring by grouping).

$$a) x^3 - 4x^2y + xy^2 - 4y^3 = x^2(x - 4y) + y^2(x - 4y) = (x^2 + y^2)(x - 4y)$$

$$b) 20x^2 - 3y^2 + 4xy^2 - 15x = 20x^2 - 15x + 4xy^2 - 3y^2 = 5x(4x - 3) + y^2(4x - 3) = (5x + y^2)(4x - 3)$$

Algebraic identities (Special products)

$$1) (a + b)^2 = a^2 + 2ab + b^2$$

$$(x + 5)^2 = x^2 + 10x + 25$$

$$2) (a - b)^2 = a^2 - 2ab + b^2$$

$$(2a - 3)^2 = 4a^2 - 12a + 9$$

$$3) (a - b)(a + b) = a^2 - b^2$$

$$(5x - 4y)(5x + 4y) = 25x^2 - 16y^2$$

$$a) 16x^2 + 40xy + 25y^2 = (4x + 5y)^2$$

$$b) 9x^2 - 12xy + 4y^2 = (3x - 2y)^2$$

$$c) \underbrace{(2x + 3y)^2}_{a^2} - \underbrace{(x - 4y)^2}_{b^2} = [\underbrace{2x + 3y}_{a} - \underbrace{(x - 4y)}_{b}][\underbrace{2x + 3y}_{a} + \underbrace{(x - 4y)}_{b}] = (a - b)(a + b)$$

$$= (2x + 3y - x + 4y)(2x + 3y + x - 4y) = (x + 7y)(3x - y)$$

Quadratic expressions written as a product of linear factors.

$$1) (x + a)(x + b) = x^2 + (a + b)x + ab$$

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

a and b are natural numbers

$$2) (x - a)(x - b) = x^2 - (a + b)x + ab$$

$$x^2 - 9x + 20 = (x - 4)(x - 5)$$

$$3) (x+a)(x-b) = x^2 + (a-b)x - ab$$

$$x^2 - 2x - 24 = (x+4)(x-6)$$

$$a) x^2 + 7x + 12 = (x+3)(x+4)$$

$$d) x^2 + 2x - 24 = (x+6)(x-4)$$

$$b) x^2 - 11x + 28 = (x-4)(x-7)$$

$$e) x^2 - 2x - 35 = (x+5)(x-7)$$

$$c) x^2 - 3x - 18 = (x+3)(x-6)$$

$$f) x^2 - 10x + 16 = (x-2)(x-8)$$

Review exercises

Factor the following expressions.

$$a) 18xy^3 - 8x^3y = 2xy(9y^2 - 4x^2) = 2xy(3y-2x)(3y+2x)$$

$$b) x^3 - 6x^2y - 2xy + 12y^2 = x^2(x-6y) - 2y(x-6y) = \\ = (x^2 - 2y)(x-6y)$$

$$c) 16x^2 - 24xy - 18x + 27y = 8x(2x-3y) - 9(2x-3y) = \\ = (8x-9)(2x-3y)$$

$$d) (x-2y)^2 - (2x-y)^2 = [x-2y-(2x-y)][x-2y+(2x-y)] = \\ = (x-2y-2x+y)(x-2y+2x-y) = (-x-y)(3x-3y) = \\ = -3(x+y)(x-y)$$

$$e) x^2 + 7x - 30 = (x+10)(x-3)$$

$$f) 4x^2 - 36 = (2x-6)(2x+6) = 4(x-3)(x+3)$$

$$g) x^2 + 10x + 25 = (x+5)^2$$

$$h) 3x^2 - 11x - 4 = 3x^2 - 12x + x - 4 = 3x(x-4) + (x-4) = (3x+1)(x-4)$$