Series: Mathematics – from zero to engineer (25).

Algebra – Review exercises.

Simplify each of the following expressions.

a) $2ab - 4ac + ba - 2cb + 3ba = 6ab - 4ac - 2cb = 2(3ab - 2ac - cb)$

b) $3x^2yz - 2x^2y + 4yxz^2 - 2x^2zy + 3z^2yx - 3zy^2 =$

$= 0x^2yz + 7z^2yx - 3zy^2 = 7z^2yx - 3zy^2 = yz(7xz - 3y)$.

c) $c^p \cdot c^{-q} : c^{-2} = c^{p+(-q)-(-2)} = c^{p-q+2}$

d) $$\frac{\left(x^{\frac{1}{2}}\right)^{-\frac{2}{3}} : \left(y^{\frac{3}{4}}\right)^{2} \cdot \left(x^{\frac{3}{5}}\right)^{-\frac{5}{3}}}{\left(x^{\frac{1}{4}}\right)^{-1} \cdot \left(y^{\frac{1}{3}}\right)^{6}} = \frac{x^{-\frac{1}{3}} : y^{\frac{3}{2}} \cdot x^{-1}}{x^{-\frac{1}{4}} \cdot y^{2}} = \frac{x^{-\frac{1}{3}} \cdot y^{-\frac{3}{2}} \cdot x^{-1}}{x^{-\frac{1}{4}} \cdot y^{2}} =$$

$= x^{-\frac{13}{12}} \cdot y^{-\frac{7}{2}}$

$-\frac{3}{2} - 2 = -\frac{7}{2}$

$-\frac{1}{3} + (-1) - \left(-\frac{1}{4}\right) = -\frac{1}{3} - 1 + \frac{1}{4} =$

$= -\frac{4}{3} + \frac{1}{4} = -\frac{16}{12} + \frac{3}{12} = -\frac{13}{12}$

Remove the brackets from each of the following expressions.

a) $2f(3g - 4h)(g + 2h) = 2f(3g^2 + 6gh - 4gh - 8h^2) = 2f(3g^2 + 2gh - 8h^2) =$

$= 6fg^2 + 4fgh - 16fh^2$

b) $(5x - 6y)(2x + 6y)(5x - y) = (10x^2 + 30xy - 12xy - 36y^2)(5x - y) =$

$= (10x^2 + 18xy - 36y^2)(5x - y) = 50x^3 - 10x^2y + 80x^2y - 18xy^2 - 180xy^2 + 36y^2 =$

$= 50x^3 + 70x^2y - 198xy^2 + 36y^2$

c) $4\{3[p - 2(q - 3)] - 3(r - 2)\} = 4\{3[p - 2q + 6] - 3r + 6\} =$

$= 4\{3p - 6q + 18 - 3r + 6\} = 4(3p - 6q + 24 - 3r) =$

$= 12p - 24q + 96 - 12r = 12p - 24q - 12r + 96$

Evaluate using a calculator or by applying the change-of-base formula where necessary.

a) $\log 0{,}0270 = -1{,}569$

b) $\ln 47{,}89 = 3{,}869$

c) $\log_7 126{,}4 = 2{,}487$

$$\log_7 10 \cdot \log_{10} 126{,}4 = \frac{1}{\log_{10} 7} \cdot \log_{10} 126{,}4 = \frac{2{,}102}{0{,}845} = 2{,}487$$

Write the following expressions in logarithmic form.

a) $V = \frac{\pi h}{4}(D-h)(D+h)$

$$\log V = \log \pi + \log h - \log 4 + \log(D-h) + \log(D+h)$$

b) $P = \frac{1}{16}(2d-1)^2 N\sqrt{S} = 2^{-4}(2d-1)^2 N\sqrt{S}$

$$\log P = \log 2^{-4} + \log(2d-1)^2 + \log N + \log\sqrt{S}$$

$$\log P = -4\log 2 + 2\log(2d-1) + \log N + \tfrac{1}{2}\log S$$

Rewrite the following expressions without using logarithms.

a) $\log x = \log P + 2\log Q - \log k - 3$

$$3 = \log P + \log Q^2 - \log k - \log x$$

$$3 = \log \frac{\frac{PQ^2}{k}}{x} \qquad 3 = \log \frac{PQ^2}{xk}$$

$$\frac{PQ^2}{xk} = 10^3$$

$$PQ^2 = 1000xk \qquad x = \frac{PQ^2}{1000k}$$

b) $\log R = 1 + \frac{1}{3}\log m + 3\log S$

$$1 = \log R - \log\sqrt[3]{m} - \log S^3$$

$$1 = \log \frac{\frac{R}{\sqrt[3]{m}}}{S^3} \qquad 1 = \log \frac{R}{S^3\sqrt[3]{m}}$$

$$\frac{R}{S^3\sqrt[3]{m}} = 10 \qquad R = 10S^3\sqrt[3]{m}$$

c) $\ln P = \frac{1}{2}\ln(Q+1) - 3\ln R + 2$

$$2 = \ln P - \ln\sqrt{Q+1} + \ln R^3$$

$$2 = \ln \frac{PR^3}{\sqrt{Q+1}} \qquad \frac{PR^3}{\sqrt{Q+1}} = e^2 \qquad PR^3 = e^2\sqrt{Q+1}$$

$$P = \frac{e^2\sqrt{Q+1}}{R^3}$$

Multiply and simplify the resulting expressions.

a) $(3x-1)(x^2-x-1) = 3x^3 - 3x^2 - 3x - x^2 + x + 1 =$

$$= 3x^3 - 4x^2 - 2x + 1.$$

b) $(a^2+2a+2)(3a^2+4a+4)=$

$$= 3a^4+4a^3+4a^2+6a^3+8a^2+8a+6a^2+8a+8=$$

$$= 3a^4+10a^3+18a^2+16a+8$$

Simplify each of the following expressions.

a) $\frac{x^3}{y^2} : \frac{x}{y^3} = \frac{x^3}{y^2} \cdot \frac{y^3}{x} = \frac{x^3y^3}{xy^2} = x^2y$

b) $\frac{ab}{c} : \frac{ac}{b} \cdot \frac{bc}{a} = \frac{ab}{c} \cdot \frac{b}{ac} \cdot \frac{bc}{a} = \frac{ab^3c}{a^2c^2} = \frac{b^3}{ac}$

Perform the division.

a) $-\{(x^2+2x-3):(x-1) = x+3$

$\phantom{-\{}x^2 - x$

$\phantom{-\{}0 \quad -\{3x-3$

$\phantom{-\{0 \quad -\{}3x-3$

$\phantom{-\{0 \quad -\{3x-}0$

$(x-1)(x+3) = x^2+2x-3$

b) $\frac{n^3+27}{n+3}$

$-\{(n^3+0n^2+0n+27):(n+3) = n^2-3n+9$

$\phantom{-\{}n^3+3n^2$

$\phantom{-\{}0 \quad -\{-3n^2+0n$

$\phantom{-\{0 \quad -\{}-3n^2-9n$

$\phantom{-\{0 \quad -\{-3n^2} -\{9n+27$

$\phantom{-\{0 \quad -\{-3n^2 -\{}9n+27$

$\phantom{-\{0 \quad -\{-3n^2 -\{9n+2}0$

$(n+3)(n^2-3n+9) = n^3+27$

c) $\frac{3a^3+2a^2+1}{a+1}$

$-\{(3a^3+2a^2+0a+1):(a+1) = 3a^2-a+1$

$\phantom{-\{}3a^3+3a^2$

$\phantom{-\{3a^3} -\{-a^2+0a$

$\phantom{-\{3a^3 -\{}-a^2-a$

$\phantom{-\{3a^3 -\{-a^2} -\{a+1$

$\phantom{-\{3a^3 -\{-a^2 -\{}a+1$

$\phantom{-\{3a^3 -\{-a^2 -\{a+}0$

$(a+1)(3a^2-a+1) = 3a^3+2a^2+1$

Factor the following expressions.

a) $36x^3y^2 - 8x^2y = 4x^2y(9xy-2)$

b) $x^3+3x^2y+2xy^2+6y^3 = x^2(x+3y)+2y^2(x+3y) = (x+3y)(x^2+2y^2)$

c) $4x^2+12x+9 = (2x+3)^2$

d) $(3x+4y)^2-(2x-y)^2=[(3x+4y-(2x-y)][3x+4y+(2x-y)]=$

$=(3x+4y-2x+y)(3x+4y+2x-y)=(x+5y)(5x+3y)$

e) $x^2+10x+24=(x+6)(x+4)$

f) $x^2-10x+16=(x-8)(x-2)$

g) $x^2-5x-36=(x+4)(x-9)$

h) $6x^2+5x-6=6x^2+9x-4x-6=3x(2x+3)-2(2x+3)=$

$=(2x+3)(3x-2).$