Series: Mathematics – from zero to engineer (28).

Polynomials – polynomial expressions.

Evaluating polynomials – Horner's method.

$$f(x)=5x^3+2x^2-3x+6.$$

1° $5x+2$

2° $(5x+2)x$

3° $(5x+2)x-3$

4° $[(5x+2)x-3]x$

5° $[(5x+2)x-3]x+6$

$$f(x)=[(5x+2)x-3]x+6$$

$$f(4)=[(5\cdot 4+2)\cdot 4-3]\cdot 4+6=[22\cdot 4-3]\cdot 4+6=85\cdot 4+6=340+6=346$$

$$f(2)=[(5\cdot 2+2)\cdot 2-3]\cdot 2+6=[12\cdot 2-3]\cdot 2+6=21\cdot 2+6=42+6=48$$

$$f(-1)=[(5\cdot(-1)+2)\cdot(-1)-3]\cdot(-1)+6=[3-3]\cdot(-1)+6=0$$

$$f(x)=3x^4+2x^2-4x+5$$

$$f(x)=3x^4+0x^3+2x^2-4x+5$$

1° $3x+0$

2° $(3x+0)x$

3° $(3x+0)x+2$

4° $[(3x+0)x+2]x$

5° $[(3x+0)x+2]x-4$

6° $\{[(3x+0)x+2]x-4\}x$

7° $\{[(3x+0)x+2]x-4\}x+5$

$$f(x)=\{[(3x+0)x+2]x-4\}x+5$$

$$f(2)=\{[(3\cdot 2+0)\cdot 2+2]\cdot 2-4\}\cdot 2+5=\{14\cdot 2-4\}\cdot 2+5=24\cdot 2+5=53$$

Write the polynomial in nested form and evaluate the polynomial f for the given value of x.

a) $f(x)=4x^3+3x^2+2x-4$, $x=2$.

$$f(x)=[(4x+3)x+2]x-4$$

$$f(2)=[(4\cdot 2+3)\cdot 2+2]\cdot 2-4=[22+2]\cdot 2-4=48-4=44$$

b) $f(x)=2x^4+x^3-3x^2+5x-6$, $x=3$

$$f(x)=\{[(2x+1)x-3]x+5\}x-6$$

$f(3) = \{[(2\cdot 3+1)\cdot 3-3]\cdot 3+5\}\cdot 3-6 = \{[21-3]\cdot 3+5\}\cdot 3-6 =$

$= \{54+5\}\cdot 3-6 = 177-6 = 171$

c) $f(x) = x^4-3x^3+2x-3$, $x=5$

$f(x) = x^4-3x^3+0x^2+2x-3$

$f(x) = \{[(x-3)x+0]x+2\}x-3$

$f(5) = \{[(5-3)\cdot 5+0]\cdot 5+2\}\cdot 5-3 = 52\cdot 5-3 = 260-3 = 257$

d) $f(x) = 2x^4-5x^3-3x^2+4$, $x=4$

$f(x) = 2x^4-5x^3-3x^2+0x+4$

$f(x) = \{[(2x-5)x-3]x+0\}x+4$

$f(4) = \{[(2\cdot 4-5)\cdot 4-3]\cdot 4+0\}\cdot 4+4 = 36\cdot 4+4 = 148$

Bezout's theorem.

$$\frac{f(x)}{x-a} = g(x) + \frac{R}{x-a} \quad / \cdot (x-a)$$

$$f(x) = (x-a)\cdot g(x) + R$$

for: $x=a$

$$f(a) = 0\cdot g(x) + R \rightarrow f(a) = R$$

If the polynomial $f(x)$ is divided by $x-a$, the remainder is $f(a)$.

- determine the remainder.

$(x^3+3x^2-13x-10):(x-3)$

$[(x+3)x-13]x-10 \rightarrow f(3) = [(3+3)\cdot 3-13]\cdot 3-10 = 15-10 = 5$

- this can be verified by performing long division.

$$
\begin{array}{l}
(x^3+3x^2-13x-10):(x-3) = x^2+6x+5 \\
-\{x^3-3x^2 \\
\quad\quad \{6x^2-13x \\
\quad\quad -\{6x^2-18x \\
\quad\quad\quad\quad \{5x-10 \\
\quad\quad\quad\quad -\{5x-15 \\
\quad\quad\quad\quad\quad\quad 5
\end{array}
$$

$(x^3+3x^2-13x-10) = (x-3)\cdot(x^2+6x+5)+5$

Determine the remainder in each of the following cases.

a) $(5x^3-4x^2-3x+6):(x-2)$

$[(5x-4)x-3]x+6 \rightarrow f(2)=[(5\cdot 2-4)\cdot 2-3]\cdot 2+6=18+6=24$

b) $(4x^3-3x^2+5x-3):(x-4)$

$[(4x-3)x+5]x-3 \rightarrow f(4)=[(4\cdot 4-3)\cdot 4+5]\cdot 4-3=57\cdot 4-3=$

$=228-3=225$

c) $(x^3-2x^2-3x+5):(x-5)$

$[(x-2)x-3]x+5 \rightarrow f(5)=[(5-2)\cdot 5-3]\cdot 5+5=60+5=65$

d) $(2x^3+3x^2-x+4):(x+2)$

$[(2x+3)x-1]x+4 \rightarrow f(-2)=[(2\cdot(-2)+3)\cdot(-2)-1]\cdot(-2)+4=-2+4=2$

e) $(3x^3-11x^2+10x-12):(x-3)$

$[(3x-11)x+10]x-12 \rightarrow f(3)=[(3\cdot 3-11)\cdot 3+10]\cdot 3-12=12-12=0$