Series: Mathematics - from zero to engineer (29).

The factor theorem.

- if the polynomial $f(x)$ is divisible by $x-a$, then the remainder of the division is $f(a)$.
- if $f(a)=0$, then $x-a$ is a factor of $f(x)$.

for example:

$$f(x)=2x^3+x^2-13x+6$$

$$(2x^3+x^2-13x+6):(x-2)$$

$$f(x)=[(2x+1)x-13]x+6 \rightarrow f(2)=[(2\cdot 2+1)\cdot 2-13]\cdot 2+6=(-3)\cdot 2+6=$$
$$=-6+6=0.$$

$$-\{(2x^3+x^2-13x+6):(x-2)=2x^2+5x-3$$
$$-\{\underline{2x^3-4x^2}$$
$$-\{\underline{5x^2-13x}$$
$$-\{\underline{5x^2-10x}$$
$$-\{\underline{-3x+6}$$
$$-\{\underline{-3x+6}$$
$$0$$

$$f(x)=2x^3+x^2-13x+6=(x-2)(2x^2+5x-3)$$

The general form of a quadratic expression.

$$f(x)=ax^2+bx+c$$

The general form of a quadratic equation.

$$ax^2+bx+c=0, \quad a\neq 0.$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

- a quadratic equation in general form has two solutions.

$$2x^2+5x-3=0$$

$$x=\frac{-5\pm\sqrt{5^2-4\cdot 2\cdot(-3)}}{2\cdot 2}=\frac{-5\pm\sqrt{25+24}}{4}=\frac{-5\pm\sqrt{49}}{4}=\frac{-5\pm 7}{4}=-3 \text{ or } \frac{1}{2}$$

$$x_1=-3, \quad x_2=\frac{1}{2}$$

remember that

$$ax^2+bx+c=a(x-x_1)(x-x_2)$$

$$2x^2 + 5x - 3 = 2(x+3)(x-\tfrac{1}{2}) = (x+3)(2x-1)$$

$$f(x) = 2x^3 + x^2 - 13x + 6 = (x-2)(2x^2+5x-3) = (x-2)(x+3)(2x-1)$$

$$f(x) = x^3 - 5x^2 - 2x + 24$$

$$(x^3 - 5x^2 - 2x + 24) : (x-3)$$

$$f(x) = [(x-5)x-2]x+24 \rightarrow f(3) = [(3-5)\cdot 3 - 2]\cdot 3 + 24 = -24 + 24 = 0$$

$$
\begin{array}{l}
(x^3 - 5x^2 - 2x + 24) : (x-3) = x^2 - 2x - 8 \\
-\{x^3 - 3x^2\} \\
\quad -2x^2 - 2x \\
\quad -\{-2x^2 + 6x\} \\
\qquad -8x + 24 \\
\qquad -\{-8x + 24\} \\
\qquad\quad 0
\end{array}
$$

$$f(x) = x^3 - 5x^2 - 2x + 24 = (x-3)(x^2-2x-8)$$

$$x^2 - 2x - 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - 4\cdot 1\cdot(-8)}}{2\cdot 1} = \frac{2 \pm \sqrt{4+32}}{2} = \frac{2\pm 6}{2} = -2 \text{ or } 4$$

$x_1 = -2$, $x_2 = 4$

$$x^2 - 2x - 8 = (x+2)(x-4)$$

$$f(x) = x^3 - 5x^2 - 2x + 24 = (x-3)(x^2-2x-8) = (x-3)(x+2)(x-4)$$

$$f(x) = x^3 - 6x^2 - 7x + 60$$

$$(x^3 - 6x^2 - 7x + 60) : (x-4)$$

$$f(x) = [(x-6)x-7]x+60 \rightarrow f(4) = [(4-6)\cdot 4 - 7]\cdot 4 + 60 = -60 + 60 = 0$$

$$
\begin{array}{l}
(x^3 - 6x^2 - 7x + 60) : (x-4) = x^2 - 2x - 15 \\
-\{x^3 - 4x^2\} \\
\quad -2x^2 - 7x \\
\quad -\{-2x^2 + 8x\} \\
\qquad -15x + 60 \\
\qquad -\{-15x + 60\} \\
\qquad\quad 0
\end{array}
$$

$$f(x) = x^3 - 6x^2 - 7x + 60 = (x-4)(x^2-2x-15)$$

$$x^2 - 2x - 15 = (x+3)(x-5) \qquad (x+a)(x-b) = x^2 + (a-b)x - ab$$

$$f(x) = x^3 - 6x^2 - 7x + 60 = (x-4)(x^2-2x-15) = (x-4)(x+3)(x-5)$$

Factoring a polynomial when the first factor is not given.

$f(x) = x^3 + 5x^2 - 2x - 24 = [(x+5)x - 2]x - 24$

- we look for a value $x = k$ such that $f(k) = 0$

$k = 1 \rightarrow [(1+5)\cdot 1 - 2]\cdot 1 - 24 = -20 \neq 0$

$k = -1 \rightarrow [(-1+5)\cdot(-1) - 2]\cdot(-1) - 24 = -18 \neq 0$

$k = 2 \rightarrow [(2+5)\cdot 2 - 2]\cdot 2 - 24 = 0$ ✓ $\quad f(2) = 0$

$$
\begin{array}{r}
(x^3 + 5x^2 - 2x - 24) : (x-2) = x^2 + 7x + 12 \\
-\{x^3 - 2x^2 \\
\hline
7x^2 - 2x \\
-\{7x^2 - 14x \\
\hline
12x - 24 \\
-\{12x - 24 \\
\hline
0
\end{array}
$$

$x^2 + 7x + 12 = (x+3)(x+4)$

$x^2 + (a+b)x + ab = (x+a)(x+b)$

$f(x) = x^3 + 5x^2 - 2x - 24 = (x-2)(x+3)(x+4)$

$f(x) = 2x^3 - 9x^2 + 7x + 6 = [(2x-9)x + 7]x + 6$

$k = 1 \rightarrow [(2\cdot 1) - 9)\cdot 1 + 7]\cdot 1 + 6 = 6 \neq 0$

$k = -1 \rightarrow [(2\cdot(-1) - 9)\cdot(-1) + 7]\cdot(-1) + 6 = -12 \neq 0$

$k = 2 \rightarrow [(2\cdot 2 - 9)\cdot 2 + 7]\cdot 2 + 6 = 0$ ✓ $\quad f(2) = 0$

$$
\begin{array}{r}
(2x^3 - 9x^2 + 7x + 6) : (x-2) = 2x^2 - 5x - 3 \\
-\{2x^3 - 4x^2 \\
\hline
-5x^2 + 7x \\
-\{-5x^2 + 10x \\
\hline
-3x + 6 \\
-\{-3x + 6 \\
\hline
0
\end{array}
$$

$(2x_3^3 - 9x_2^2 + 7x + 6) = (x-2)(2x^2 - 5x - 3)$

$2x^2 - 5x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{(-5)^2 - 4\cdot 2\cdot(-3)}}{2\cdot 2} = \frac{5 \pm \sqrt{25 + 24}}{4} =$$

$$= \frac{5 \pm 7}{4} = 3 \text{ or } -\frac{1}{2} \qquad x_1 = 3 \,;\, x_2 = -\frac{1}{2}$$

$2x^2 - 5x - 3 = 2(x-3)(x+\frac{1}{2}) = (x-3)(2x+1)$

$(2x_3^3 - 9x_2^2 + 7x + 6) = (x-2)(2x^2 - 5x - 3) = (x-2)(x-3)(2x+1)$