Series: Mathematics – from zero to engineer (30).

Factoring fourth-degree polynomials.

$$f(x) = 2x^4 - x^3 - 8x^2 + x + 6 = \{[(2x-1)x - 8]x + 1\}x + 6$$

for $x = 1$ $\quad \{[(2\cdot 1 - 1)\cdot 1 - 8]\cdot 1 + 1\}\cdot 1 + 6 = 0 \rightarrow f(1) = 0$

$$\begin{array}{l}
(2x^4 - x^3 - 8x^2 + x + 6) : (x-1) = 2x^3 + x^2 - 7x - 6 \\
-(2x^4 - 2x^3) \\
\quad x^3 - 8x^2 \\
\quad -(x^3 - x^2) \\
\qquad -7x^2 + x \\
\qquad -(-7x^2 + 7x) \\
\qquad\quad -6x + 6 \\
\qquad\quad -(-6x + 6) \\
\qquad\qquad 0
\end{array}$$

$$\underbrace{2x^4 - x^3 - 8x^2 + x + 6}_{f(x)} = (x-1)\underbrace{(2x^3 + x^2 - 7x - 6)}_{g(x)}$$

$$g(x) = 2x^3 + x^2 - 7x - 6 = [(2x+1)x - 7]x - 6$$

for $x = -1$ $\quad [(2\cdot(-1) + 1)\cdot(-1) - 7]\cdot(-1) - 6 = 0 \rightarrow f(-1) = 0$

$$\begin{array}{l}
(2x^3 + x^2 - 7x - 6) : (x+1) = 2x^2 - x - 6 \\
-(2x^3 + 2x^2) \\
\quad -x^2 - 7x \\
\quad -(-x^2 - x) \\
\qquad -6x - 6 \\
\qquad -(-6x - 6) \\
\qquad\quad 0
\end{array}$$

$$2x^4 - x^3 - 8x^2 + x + 6 = (x-1)(x+1)(2x^2 - x - 6)$$

$$2x^2 - x - 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{(-1)^2 - 4\cdot 2\cdot(-6)}}{2\cdot 2} = \frac{1 \pm \sqrt{1 + 48}}{4} = \frac{1 \pm 7}{4} = 2 \text{ or } -\frac{3}{2}$$

$x_1 = 2$, $x_2 = -\frac{3}{2}$

$$2x^2 - x - 6 = 2(x-2)\left(x + \frac{3}{2}\right) = (x-2)(2x+3)$$

$$2x^4 - x^3 - 8x^2 + x + 6 = (x-1)(x+1)(x-2)(2x+3)$$

$$f(x) = 2x^4 - 5x^3 - 15x^2 + 10x + 8 = \{[(2x-5)x - 15]x + 10\}x + 8$$

for $x = 1$ $\quad \{[(2\cdot 1 - 5)\cdot 1 - 15]\cdot 1 + 10\}\cdot 1 + 8 = 0 \rightarrow f(1) = 0$

for $x = -2$ $\quad \{[(2\cdot(-2) - 5)\cdot(-2) - 15]\cdot(-2) + 10\}\cdot(-2) + 8 = 0 \rightarrow f(-2) = 0$

$$2x^4 - 5x^3 - 15x^2 + 10x + 8 = (x-1)(x+2)(ax^2 + bx + c)$$

$$\Downarrow$$

$$(x^2 + x - 2)$$

$$(2x^4 - 5x^3 - 15x^2 + 10x + 8) : (x^2 + x - 2) = 2x^2 - 7x - 4$$

$$\begin{array}{l}
-\left\{\begin{array}{l} 2x^4 - 5x^3 - 15x^2 + 10x + 8 \\ 2x^4 + 2x^3 - 4x^2 \end{array}\right. \\
\qquad -\left\{\begin{array}{l} -7x^3 - 11x^2 + 10x \\ -7x^3 - 7x^2 + 14x \end{array}\right. \\
\qquad\qquad -\left\{\begin{array}{l} -4x^2 - 4x + 8 \\ -4x^2 - 4x + 8 \end{array}\right. \\
\qquad\qquad\qquad\qquad 0
\end{array}$$

$$2x^2 - 7x - 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{(-7)^2 - 4 \cdot 2 \cdot (-4)}}{2 \cdot 2} = \frac{7 \pm \sqrt{49 + 32}}{4} =$$

$$= \frac{7 \pm 9}{4} = 4 \text{ or } -\frac{1}{2} \qquad x_1 = 4 \;,\; x_2 = -\frac{1}{2}$$

$$2x^2 - 7x - 4 = 2(x-4)\left(x + \frac{1}{2}\right) = (x-4)(2x+1)$$

$$2x^4 - 5x^3 - 15x^2 + 10x + 8 = (x-1)(x+2)(x-4)(2x+1)$$

$$f(x) = 2x^4 - 3x^3 - 14x^2 + 33x - 18 = \{[(2x-3)x - 14]x + 33\}x - 18$$

for $x = 1$ $\quad \{[(2 \cdot 1 - 3) \cdot 1 - 14] \cdot 1 + 33\} \cdot 1 - 18 = 0 \rightarrow f(1) = 0$

for $x = 2$ $\quad \{[(2 \cdot 2 - 3) \cdot 2 - 14] \cdot 2 + 33\} \cdot 2 - 18 = 0 \rightarrow f(2) = 0$

$$2x^4 - 3x^3 - 14x^2 + 33x - 18 = \underbrace{(x-1)(x-2)}_{\Downarrow \; (x^2 - 3x + 2)}(ax^2 + bx + c)$$

$$(2x^4 - 3x^3 - 14x^2 + 33x - 18) : (x^2 - 3x + 2) = 2x^2 + 3x - 9$$

$$\begin{array}{l}
-\left\{\begin{array}{l} 2x^4 - 3x^3 - 14x^2 + 33x - 18 \\ 2x^4 - 6x^3 + 4x^2 \end{array}\right. \\
\qquad -\left\{\begin{array}{l} 3x^3 - 18x^2 + 33x \\ 3x^3 - 9x^2 + 6x \end{array}\right. \\
\qquad\qquad -\left\{\begin{array}{l} -9x^2 + 27x - 18 \\ -9x^2 + 27x - 18 \end{array}\right. \\
\qquad\qquad\qquad\qquad 0
\end{array}$$

$$2x^2 + 3x - 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot (-9)}}{2 \cdot 2} = \frac{-3 \pm \sqrt{9 + 72}}{4} =$$

$$= \frac{-3 \pm 9}{4} = -3 \text{ or } \frac{3}{2} \qquad x_1 = -3 \;,\; x_2 = \frac{3}{2}$$

$$2x^2 + 3x - 9 = 2(x+3)\left(x - \frac{3}{2}\right) = (x+3)(2x-3)$$

$$2x^4 - 3x^3 - 14x^2 + 33x - 18 = (x-1)(x-2)(x+3)(2x-3)$$

Review exercises.

Write $f(x) = 6x^3 - 5x^2 + 4x - 3$ in nested form and calculate $f(2)$.

$$[(6x-5)x+4]x-3$$

$$f(2) = [(6\cdot 2 - 5)\cdot 2 + 4]\cdot 2 - 3 = 33$$

Without performing the entire division, determine the remainder.

$$(x^4 - 2x^3 + 3x^2 - 4) : (x-2)$$

$$x^4 - 2x^3 + 3x^2 + 0x - 4 = \{[(x-2)x+3]x+0\}x-4$$

$$f(2) = \{[(2-2)\cdot 2 + 3]\cdot 2 + 0\}\cdot 2 - 4 = 8 \rightarrow \text{remainder}$$

Factorise the polynomial $6x^4 + x^3 - 25x^2 - 4x + 4$.

$$6x^4 + x^3 - 25x^2 - 4x + 4 = \{[(6x+1)x-25]x-4\}x+4$$

for $x = 2$ $\quad \{[(6\cdot 2+1)\cdot 2 - 25]\cdot 2 - 4\}\cdot 2 + 4 = 0 \rightarrow f(2) = 0$

for $x = -2$ $\quad (6\cdot(-2)+1)\cdot(-2)-25]\cdot(-2)-4\}\cdot(-2)+4 = 0 \rightarrow f(-2) = 0$

$$6x^4 + x^3 - 25x^2 - 4x + 4 = \underbrace{(x-2)(x+2)}_{(x^2-4)}(ax^2+bx+c)$$

$$
\begin{array}{rrrrrl}
 & (6x^4 & + x^3 & - 25x^2 & - 4x & + 4) : (x^2-4) = 6x^2 + x - 1 \\
- & 6x^4 & + 0 & - 24x^2 & & \\
\hline
 & & x^3 & - x^2 & - 4x & \\
- & & x^3 & + 0 & - 4x & \\
\hline
 & & & -x^2 & + 0 & + 4 \\
- & & & -x^2 & + 0 & + 4 \\
\hline
 & & & & & 0
\end{array}
$$

$$6x^2 + x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4\cdot 6\cdot(-1)}}{2\cdot 6} = \frac{-1 \pm \sqrt{1+24}}{12} =$$

$$= \frac{-1 \pm 5}{12} = -\frac{1}{2} \text{ or } \frac{1}{3} \qquad x_1 = -\frac{1}{2}\ ,\ x_2 = \frac{1}{3}$$

$$6x^2 + x - 1 = 6\left(x+\frac{1}{2}\right)\left(x-\frac{1}{3}\right) = 2\left(x+\frac{1}{2}\right)3\left(x-\frac{1}{3}\right) = (2x+1)(3x-1)$$

$$6x^4 + x^3 - 25x^2 - 4x + 4 = (x-2)(x+2)(2x+1)(3x-1)$$