Series: Mathematics – from zero to engineer (31).

## Review exercises.

Calculate correct to three significant figures.

$$\sqrt{\frac{2V}{\pi h} - \frac{h^2}{3}}$$

a) $V = 23{,}05$ , $h = 2{,}69$

$$\sqrt{\frac{2V}{\pi h} - \frac{h^2}{3}} = \sqrt{\frac{2 \cdot 23{,}05}{3{,}14 \cdot 2{,}69} - \frac{2{,}69^2}{3}} = \sqrt{\frac{46{,}1}{8{,}45} - \frac{7{,}24}{3}} = \sqrt{5{,}46 - 2{,}41} =$$

$$= \sqrt{3{,}05} = 1{,}75$$

b) $V = 85{,}67$ , $h = 5{,}44$

$$\sqrt{\frac{2V}{\pi h} - \frac{h^2}{3}} = \sqrt{\frac{2 \cdot 85{,}67}{3{,}14 \cdot 5{,}44} - \frac{5{,}44^2}{3}} = \sqrt{\frac{171{,}34}{17{,}1} - \frac{29{,}6}{3}} = \sqrt{10{,}02 - 9{,}87} =$$

$$= \sqrt{0{,}15} = 0{,}387$$

Given $q = \sqrt{(5p^2 - 1)^2 - 2}$ , express $p$ in terms of $q$.

$q = \sqrt{(5p^2 - 1)^2 - 2} \quad /^2$

$q^2 = (5p^2 - 1)^2 - 2$

$q^2 + 2 = (5p^2 - 1)^2 \quad / \sqrt{\ }$

$\sqrt{q^2 + 2} = 5p^2 - 1$

$\sqrt{q^2 + 2} + 1 = 5p^2 \quad / : 5$

$\frac{\sqrt{q^2 + 2} + 1}{5} = p^2 \quad / \sqrt{\ }$

$$p = \sqrt{\frac{\sqrt{q^2 + 2} + 1}{5}}$$

Express each of the following polynomials in the nested form characteristic of Horner's method and determine the value of the polynomial for the specified values of $x$.

a) $f(x) = 2x^3 - 3x^2 + 5x - 4 = [(2x - 3)x + 5]x - 4$

$f(3) = [(2 \cdot 3 - 3) \cdot 3 + 5] \cdot 3 - 4 = 38$

b) $f(x) = 4x^3 + 2x^2 - 7x - 2 = [(4x + 2)x - 7]x - 2$

$f(2) = [(4 \cdot 2 + 2) \cdot 2 - 7] \cdot 2 - 2 = 24$

c) $f(x) = 3x^3 - 4x^2 + 8 = 3x^3 - 4x^2 + 0x + 8 = [(3x - 4)x + 0]x + 8$

$f(4) = [(3 \cdot 4 - 4) \cdot 4 + 0] \cdot 4 + 8 = 136$

d) $f(x) = 2x^3 + x - 5 = 2x^3 + 0x^2 + x - 5 = [(2x+0)x+1]x-5$

$f(5) = [(2\cdot 5+0)\cdot 5+1]\cdot 5 - 5 = 250$

Determine the remainder obtained from the division.

$(4x^3 - 5x^2 + 7x - 3) : (x-3)$

$[(4x-5)x+7]x-3 \rightarrow f(3) = [(4\cdot 3-5)\cdot 3+7]\cdot 3-3 = 81 \rightarrow$ remainder

Check whether $x-2$ is a factor of the polynomial $f(x)$.

If so, factorise $f(x)$.

$f(x) = 2x^3 + 2x^2 - 17x + 10 = [(2x+2)x-17]x+10$

$f(2) = [(2\cdot 2+2)\cdot 2-17]\cdot 2+10 = 0$

$-\{(2x^3 + 2x^2 - 17x + 10) : (x-2) = 2x^2 + 6x - 5$

$-\{2x^3 - 4x^2$

$\underline{\quad}$

$-\{6x^2 - 17x$

$-\{6x^2 - 12x$

$\underline{\quad}$

$-5x + 10$

$-5x + 10$

$\underline{\quad}$

$0$

$(2x^3 + 2x^2 - 17x + 10) = (x-2)(2x^2 + 6x - 5)$

Write $f(x) = 2x^3 + 7x^2 - 14x - 40$ as a product of three linear factors.

$f(x) = 2x^3 + 7x^2 - 14x - 40 = [(2x+7)x-14]x-40$

for $x = -2 \rightarrow f(-2) = [(2\cdot(-2)+7)\cdot(-2)-14]\cdot(-2)-40 = 0$

$-\{(2x^3 + 7x^2 - 14x - 40) : (x+2) = 2x^2 + 3x - 20$

$-\{2x^3 + 4x^2$

$\underline{\quad}$

$-\{3x^2 - 14x$

$-\{3x^2 + 6x$

$\underline{\quad}$

$-\{-20x - 40$

$-\{-20x - 40$

$\underline{\quad}$

$0$

$(2x^3 + 7x^2 - 14x - 40) = (x+2)(2x^2 + 3x - 20)$

$2x^2 + 3x - 20 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-3 \pm \sqrt{3^2-4\cdot 2\cdot(-20)}}{2\cdot 2} = \frac{-3 \pm \sqrt{9+160}}{4} =$

$= \frac{-3 \pm 13}{4} = -4 \text{ or } \frac{5}{2} \qquad x_1 = -4 \,,\; x_2 = \frac{5}{2}$

$2x^2 + 3x - 20 = 2(x+4)(x-\frac{5}{2}) = (x+4)(2x-5)$

$(2x^3 + 7x^2 - 14x - 40) = (x+2)(x+4)(2x-5)$

Express $f(x)=2x^4-7x^3-2x^2+13x+6$ as a product of four linear factors.

$f(x)=2x^4-7x^3-2x^2+13x+6=\{[(2x-7)x-2]x+13\}x+6$

for $x=-1 \rightarrow f(-1)=\{[(2\cdot(-1)-7)\cdot(-1)-2]\cdot(-1)+13\}\cdot(-1)+6=0$

for $x=2 \rightarrow f(2)=\{[(2\cdot 2-7)\cdot 2-2]\cdot 2+13\}\cdot 2+6=0$

$2x^4-7x^3-2x^2+13x+6=(x+1)(x-2)(ax^2+bx+c)$

$\Downarrow$

$(x^2-x-2)$

$-\{(2x^4-7x^3-2x^2+13x+6):(x^2-x-2)=2x^2-5x-3$

$-\{2x^4-2x^3-4x^2$

$-5x^3+2x^2+13x$

$-\{-5x^3+5x^2+10x$

$-3x^2+3x+6$

$-\{-3x^2+3x+6$

$0$

$2x^4-7x^3-2x^2+13x+6=(x+1)(x-2)(2x^2-5x-3)$

$2x^2-5x-3=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{5\pm\sqrt{(-5)^2-4\cdot 2\cdot(-3)}}{2\cdot 2}=\frac{5\pm\sqrt{25+24}}{4}=$

$=\frac{5\pm 7}{4}=-\frac{1}{2}$ or $3 \quad x_1=-\frac{1}{2}\ ;\ x_2=3$

$2x^2-5x-3=2(x+\frac{1}{2})(x-3)=(2x+1)(x-3)$

$2x^4-7x^3-2x^2+13x+6=(x+1)(x-2)(2x+1)(x-3)$