

Series: Mathematics - from zero to engineer (4)

Review exercises

1) write each of the following numbers as a product of prime factors.

$$\begin{array}{r|l} 429 & 3 \\ 143 & 11 \\ 13 & 13 \\ 1 & \end{array}$$

$$429 = 3 \cdot 11 \cdot 13$$

$$\begin{array}{r|l} 1820 & 2 \\ 910 & 2 \\ 455 & 5 \\ 91 & 7 \\ 13 & 13 \\ 1 & \end{array}$$

$$1820 = 2 \cdot 2 \cdot 5 \cdot 7 \cdot 13$$

$$\begin{array}{r|l} 2992 & 2 \\ 1496 & 2 \\ 748 & 2 \\ 374 & 2 \\ 187 & 11 \\ 17 & 17 \\ 1 & \end{array}$$

$$2992 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 11 \cdot 17$$

$$\begin{array}{r|l} 3185 & 5 \\ 637 & 7 \\ 91 & 7 \\ 13 & 13 \\ 1 & \end{array}$$

$$3185 = 5 \cdot 7 \cdot 7 \cdot 13$$

2) find the GCD and LCM of the following pairs of numbers

a) 63, 42

$$\begin{array}{r|l} 63 & 3 \checkmark \\ 21 & 3 \checkmark \\ 7 & 7 \checkmark \\ 1 & \end{array}$$

$$\begin{array}{r|l} 42 & 2 \checkmark \\ 21 & 3 - \\ 7 & 7 - \\ 1 & \end{array}$$

$$\text{GCD}(63, 42) = 3 \cdot 7 = 21$$

$$\text{LCM}(63, 42) = 3 \cdot 3 \cdot 7 \cdot 2 = 126$$

b) 34, 92

$$\begin{array}{r|l} 34 & 2 \checkmark \\ 17 & 17 \checkmark \\ 1 & \end{array}$$

$$\begin{array}{r|l} 92 & 2 - \\ 46 & 2 \checkmark \\ 23 & 23 \checkmark \\ 1 & \end{array}$$

$$\text{GCD}(34, 92) = 2$$

$$\text{LCM}(34, 92) = 2 \cdot 2 \cdot 17 \cdot 23 = 1564$$

Fractions (rational numbers)

- fractions are written as an integer divided by a non-zero integer and are called rational number

$-\frac{8}{11}$ proper fraction

$\frac{12}{5}$ improper fraction

$6\frac{2}{3}$ mixed number

$\frac{4}{7}$ numerator
denominator

Multiplying fractions

$$\frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$$

$$\frac{5}{9} \cdot \frac{2}{7} = \frac{5 \cdot 2}{9 \cdot 7} = \frac{10}{63}$$

$$\frac{1}{3} \text{ of } \frac{2}{5} = \frac{1}{3} \cdot \frac{2}{5} = \frac{2}{15}$$

$$\frac{3}{8} \text{ of } \frac{5}{7} = \frac{3}{8} \cdot \frac{5}{7} = \frac{15}{56}$$

Equivalent fractions

$$\frac{4}{5} = \frac{4 \cdot 3}{5 \cdot 3} = \frac{4}{5} \cdot \frac{3}{3} = \frac{4}{5} \cdot 1 = \frac{4}{5} \rightarrow \frac{4}{5} \cdot \frac{3}{3} = \frac{12}{15}$$

therefore $\frac{4}{5} = \frac{12}{15}$

$$\frac{7}{5} \cdot \frac{4}{4} = \frac{28}{20} \quad \text{therefore} \quad \frac{7}{5} = \frac{28}{20}$$

Simplifying fractions

$$\frac{16}{96} = \frac{\cancel{16}^1}{6 \cdot \cancel{16}_1} = \frac{1}{6}$$

$$\frac{84}{108} = \frac{7 \cdot \cancel{12}^1}{9 \cdot \cancel{12}_1} = \frac{7}{9}$$

Dividing fractions

$$\frac{2}{3} : \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5} = \frac{14}{15}$$

$$\frac{7}{13} : \frac{3}{4} = \frac{7}{13} \cdot \frac{4}{3} = \frac{28}{39}$$

- to divide two fractions, multiply the first fraction by the reciprocal of the second fraction

Adding and subtracting fractions

$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$$

common denominator

$$\frac{2}{3} + \frac{1}{5} = \frac{2}{3} \cdot \frac{5}{5} + \frac{1}{5} \cdot \frac{3}{3} = \frac{10}{15} + \frac{3}{15} = \frac{13}{15}$$

no common denominator

finding a common denominator

common denominator

$$\frac{5}{9} + \frac{1}{6} = \frac{5}{9} \cdot \frac{2}{2} + \frac{1}{6} \cdot \frac{3}{3} = \frac{10}{18} + \frac{3}{18} = \frac{13}{18}$$

$$\frac{11}{15} - \frac{2}{3} = \frac{11}{15} - \frac{2}{3} \cdot \frac{5}{5} = \frac{11}{15} - \frac{10}{15} = \frac{1}{15}$$