Series: Mathematics - from zero to engineer (5).

Ratios

- a ratio is formed by the numerators of fractions with the same denominator

Write the given proportion as a ratio:

$\frac{3}{4}$ of quantity A, $\frac{1}{6}$ of quantity B, $\frac{1}{12}$ of quantity C

A: $\frac{3}{4}\cdot\frac{3}{3}=\frac{9}{12}$ B: $\frac{1}{6}\cdot\frac{2}{2}=\frac{2}{12}$ C: $\frac{1}{12}$

$9:2:1$

Percentages

- the numerator of a fraction with a denominator of 100 is called a percentage

$$\frac{5}{100}=5\% \qquad \frac{13}{100}=13\%$$

$$\frac{12}{25}=\left(\frac{12}{25}\cdot 100\right)\%=\left(\frac{12}{\cancel{25}_{1}}\cdot \cancel{25}^{1}\cdot 4\right)\%=(12\cdot 4)\%=48\%$$

- a fraction can be converted into a percentage by multiplying it by 100

How do you find a percentage of a given quantity?

$$24\%\text{ of }75=\frac{24}{100}\text{ of }75=\frac{24}{100}\cdot 75=\frac{24}{\cancel{25}\cdot 4}\cdot\cancel{25}\cdot 3=\frac{24}{4}\cdot 3=6\cdot 3=18 \quad (\frac{24}{4}\Rightarrow 6)$$

$$8\%\text{ of }25=\frac{8}{100}\text{ of }25=\frac{8}{100}\cdot 25=\frac{8}{4\cdot\cancel{25}}\cdot\cancel{25}=\frac{8}{4}=2$$

Review exercises

1) Simplify each of the following fractions to its lowest terms.

a) $\frac{24}{30}=\frac{4\cdot\cancel{6}}{5\cdot\cancel{6}}=\frac{4}{5}$ b) $\frac{72}{15}=\frac{24\cdot\cancel{3}}{5\cdot\cancel{3}}=\frac{24}{5}$ c) $-\frac{52}{65}=-\frac{4\cdot\cancel{13}}{5\cdot\cancel{13}}=-\frac{4}{5}$

d) $\frac{32}{8}=\frac{4\cdot\cancel{8}}{\cancel{8}}=\frac{4}{1}=4$

2) Evaluate the following expressions.

a) $\frac{5}{9}\cdot\frac{2}{5}=\frac{\cancel{5}\cdot 2}{9\cdot\cancel{5}}=\frac{2}{9}$ b) $\frac{13}{25}:\frac{2}{15}=\frac{13}{\cancel{25}_{5}}\cdot\frac{\cancel{15}^{3}}{2}=\frac{39}{10}=3\frac{9}{10}$

c) $\frac{5}{9}+\frac{3}{14}=\frac{5\cdot 14+3\cdot 9}{9\cdot 14}=\frac{70+27}{126}=\frac{97}{126}$

d) $\frac{3}{8} - \frac{2}{5} = \frac{3\cdot 5 - 2\cdot 8}{40} = \frac{15-16}{40} = -\frac{1}{40}$

e) $\frac{12}{7} \cdot \left(-\frac{3}{5}\right) = -\frac{12\cdot 3}{7\cdot 5} = -\frac{36}{35} = -1\frac{1}{35}$

f) $\left(-\frac{3}{4}\right) : \left(-\frac{12}{7}\right) = \frac{\cancel{3}^{1}}{4} \cdot \frac{7}{\cancel{12}_{4}} = \frac{7}{16}$

g) $\frac{19}{2} + \frac{7}{4} = \frac{19}{2}\cdot\frac{2}{2} + \frac{7}{4} = \frac{38}{4} + \frac{7}{4} = \frac{45}{4} = 11\frac{1}{4}$

h) $\frac{1}{4} - \frac{3}{8} = \frac{1}{4}\cdot\frac{2}{2} - \frac{3}{8} = \frac{2}{8} - \frac{3}{8} = -\frac{1}{8}$

3) Write the following proportions as ratios:

a) $\frac{1}{2}$ of quantity A , $\frac{2}{5}$ of quantity B , $\frac{1}{10}$ of quantity C

A: $\frac{1}{2}\cdot\frac{5}{5} = \frac{5}{10}$ B: $\frac{2}{5}\cdot\frac{2}{2} = \frac{4}{10}$ C: $\frac{1}{10}$

5 : 4 : 1

b) $\frac{1}{3}$ of quantity P , $\frac{1}{5}$ of quantity Q , $\frac{1}{4}$ of quantity R , remainder S

P: $\frac{1}{3}\cdot\frac{20}{20} = \frac{20}{60}$ Q: $\frac{1}{5}\cdot\frac{12}{12} = \frac{12}{60}$ R: $\frac{1}{4}\cdot\frac{15}{15} = \frac{15}{60}$

$20 + 12 + 15 = 47 \xrightarrow[S]{+13} 60$ S: $\frac{13}{60}$

20 : 12 : 15 : 13

4) Complete the following:

a) $\frac{2}{5} = \left(\frac{2}{5}\cdot 100\right)\% = \left(\frac{2}{\cancel{5}}\cdot\cancel{5}\cdot 20\right)\% = 40\%$

b) 58% of $25 = \frac{58}{100}\cdot 25 = \frac{58}{4\cdot\cancel{25}}\cdot\cancel{25} = \frac{58}{4} = \frac{29}{2} = 14\frac{1}{2}$

c) $\frac{7}{12} = \left(\frac{7}{12}\cdot 100\right)\% = \left(\frac{7}{3\cdot\cancel{4}}\cdot\cancel{4}\cdot 25\right)\% = \frac{7\cdot 25}{3}\% = \frac{175}{3}\% = 58\frac{1}{3}\%$

d) 17% of $50 = \frac{17}{100}\cdot 50 = \frac{17}{2\cdot\cancel{50}}\cdot\cancel{50} = \frac{17}{2} = 8\frac{1}{2}$