

Series: Mathematics - from zero to engineer (8)

Fractional exponents and roots.

- roots can be expressed as powers with fractional (rational) exponents.

$$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$$

$$6^{\frac{1}{5}} = \sqrt[5]{6} = 1,431$$

$$4^{\frac{1}{2}} = \sqrt{4} = \pm 2$$

$$81^{\frac{1}{4}} = \sqrt[4]{81} = \pm 3$$

$$(-32)^{\frac{1}{5}} = \sqrt[5]{-32} = -2$$

$$16^{\frac{1}{4}} = \sqrt[4]{16} = 1,486$$

$$8^{\frac{1}{2}} = \sqrt{8} = 2,828$$

$$19^{\frac{1}{4}} = \sqrt[4]{19} = \pm 2,088$$

$\sqrt{-4} \rightarrow$ the square root of a negative number does not exist (in the set of real numbers)

Multiplying and dividing whole numbers by powers of 10.

$$0,012045 \cdot 10^4 = 120,45 \quad (\text{four places to the right})$$

$$13,5074 \cdot 10^{-3} = 0,0135074 \quad (\text{three places to the left})$$

$$144,032 : 10^5 = 144,032 \cdot 10^{-5} = 0,00144032 \quad (\text{five places to the left})$$

$$0,012045 : 10^{-2} = 0,012045 \cdot 10^2 = 1,2045 \quad (\text{two places to the right})$$

The order of operations

exponentiation $\rightarrow$ multiplication/division $\rightarrow$ addition/subtraction

$$5(3 \cdot 4^2 : 6 - 7) = 5(3 \cdot 16 : 6 - 7) = 5(48 : 6 - 7) = 5(8 - 7) = 5$$

Scientific notation

- a decimal number in the interval $<1, 10)$, called the mantissa, multiplied by 10 raised to the appropriate power

$$0,000485 = 4,85 \cdot 10^{-4}$$

$$423,8 = 4,238 \cdot 10^2$$

$$52674 = 5,2674 \cdot 10^4$$

$$0,00723 = 7,23 \cdot 10^{-3}$$

$$0,0582 = 5,82 \cdot 10^{-2}$$

$$1523800 = 1,523800 \cdot 10^6$$

Multiplying and dividing numbers in scientific notation.

$$\begin{aligned} \text{a) } 472,3 \cdot 0,000564 &= (4,723 \cdot 10^2) \cdot (5,64 \cdot 10^{-4}) \\ &= 26,6377 \cdot 10^{-2} \\ &= 2,66377 \cdot 10 \cdot 10^{-2} \\ &= 2,6638 \cdot 10^{-1} \end{aligned}$$

- correct to four decimal places

$$\begin{aligned} \text{b) } 752000 : 0,862 &= (7,52 \cdot 10^5) : (8,62 \cdot 10^{-1}) \\ &= 0,87238 \cdot 10^5 : 10^{-1} \quad (10^5 \cdot 10^1) \\ &= 0,87238 \cdot 10^6 \\ &= 8,7238 \cdot 10^{-1} \cdot 10^6 \\ &= 8,7238 \cdot 10^5 \end{aligned}$$

- correct to four decimal places

Adding and subtracting numbers in scientific notation.

$$\begin{aligned} \text{a) } 43,6 \cdot 10^2 + 8,12 \cdot 10^3 \\ &= 4,36 \cdot 10^3 + 8,12 \cdot 10^3 \\ &= (4,36 + 8,12) \cdot 10^3 \\ &= 12,48 \cdot 10^3 \\ &= 1,248 \cdot 10^4 \end{aligned}$$

$$\begin{aligned} \text{b) } 7,84 \cdot 10^5 - 12,36 \cdot 10^3 \\ &= 7,84 \cdot 10^5 - 0,1236 \cdot 10^5 \\ &= (7,84 - 0,1236) \cdot 10^5 \\ &= 7,7164 \cdot 10^5 \end{aligned}$$

$$\begin{aligned} \text{c) } 4,25 \cdot 10^{-3} + 1,74 \cdot 10^{-2} \\ &= 0,425 \cdot 10^{-2} + 1,74 \cdot 10^{-2} \\ &= 2,165 \cdot 10^{-2} \end{aligned}$$